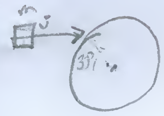
EUF 2011 - 2Sem

25/03/2016
15:00

Q1.

a) 

$\tau = I \cdot \alpha$

$a_c = \frac{v^2}{R}$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$$\boxed{v_{cm} = \frac{\sum m_i v_i}{\sum m_i}} \Rightarrow (M+m)v_{cm} = mv$$

$$\boxed{v_{cm} = \frac{mv}{M+m}}$$

b) $L_i = L_f$ just before hits

$mvR = I\omega_f \Leftrightarrow mvR = \omega_f\left(\frac{1}{2}MR^2 + mR^2\right)$

$I = \frac{1}{2}MR^2 + mR^2$

$\Downarrow$

$$\omega_f = \frac{mv\cancel{R}}{\frac{1}{2}R^{\cancel{2}}(M+m)} = \frac{2v}{R}\left(\frac{1}{1+\frac{M}{m}}\right)$$

c) $E_i = \frac{mv^2}{2}$?

$E_f = \frac{I \cdot \omega_f^2}{2}$ $\Rightarrow$ $\Delta E = \frac{I\omega_f^2}{2} - \frac{mv^2}{2} =$

Q2. a) 

$z = r \cdot \cot g\,\alpha$ $\quad x = r\cos\theta$ $\quad y = r\sin\theta$

$\dot{z} = \cot g\,\alpha \cdot \dot{r}$ $\quad \dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$ $\quad \dot{y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$

$$\left.\begin{aligned}\dot{x}^2 &= \dot{r}^2\cos^2\theta - 2\dot{r}r\dot{\theta}\cos\theta\sin\theta + r^2\dot{\theta}^2\sin^2\theta \\ \dot{y}^2 &= \dot{r}^2\sin^2\theta + 2\dot{r}r\dot{\theta}\cos\theta\sin\theta + r^2\dot{\theta}^2\cos^2\theta\end{aligned}\right\} \dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{r}^2\cot g^2\alpha\right) = \boxed{\frac{1}{2}m\dot{r}^2\operatorname{cossec}^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2}$$

$$\boxed{U = mgz = mgr\cot g\,\alpha}$$

b) $L = T - U = \frac{m\dot{r}^2}{2} \cdot \csc^2\alpha + \frac{1}{2} m r^2 \dot{\theta}^2 - mgr\cot\alpha$

for r:

$$\cancel{m}\ddot{r}\csc^2\alpha - \cancel{m}r\dot{\theta}^2 + \cancel{m}g\cot\alpha = 0$$

$$\ddot{r} - r\dot{\theta}^2\sin^2\alpha + g\cos\alpha \cdot \sin\alpha = 0$$

for θ:

$$mr^2\ddot{\theta} - 0 = 0 \Rightarrow mr^2\ddot{\theta} = 0 \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = \text{constant}$$

c) Since, θ variable cyclic is conserved

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = \text{constant}$$

$$\Downarrow$$

$mr^2\omega = mrv = L$; maybe energy because $\frac{\partial L}{\partial t} = 0$

d) $H = \sum p_i \cdot \dot{q}_i - L$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}\csc^2\alpha \Rightarrow \dot{r} = \frac{p_r \sin^2\alpha}{m}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \Rightarrow \quad \dot{\theta} = \frac{p_\theta}{mr^2}$$

$$H = \left(p_r \cdot \dot{r} + p_\theta \cdot \dot{\theta}\right) - \frac{m\dot{r}^2}{2}\csc^2\alpha - \frac{1}{2}mr^2\dot{\theta}^2 + mgr\cot\alpha$$

$$H = \frac{p_r^2 \sin^2\alpha}{m} + \frac{p_\theta^2}{mr^2} - \frac{\cancel{m} p_r^2 \sin^2\alpha}{m^2 2} - \frac{1}{2}\frac{\cancel{m}r^2 \cdot p_\theta^2}{m^2 r^{\cancel{4}2}} + mgr\cot\alpha$$

$$\boxed{= \frac{p_r^2\sin^2\alpha}{2m} + \frac{p_\theta^2}{2mr^2} + mgr\cot\alpha}$$

$$\dot{p}_r = \frac{\partial H}{\partial r} = \frac{p_\theta^2}{mr^3} - mg\cot\alpha \qquad \dot{\theta} = \frac{p_\theta}{mr^2}$$

$$\dot{p}_\theta = -\frac{\partial H}{\partial \theta} = 0 \qquad \dot{r} = \frac{\partial H}{\partial p_r} = \frac{p_r \sin^2\alpha}{m}$$

$E_m = \frac{m\dot{r}^2}{2}\csc^2\alpha + \frac{1}{2}mr^2\dot{\theta}^2 + mgr\cot\alpha$

e) oscilações radiais

$V = \frac{1}{2}kr^2$

$\dot{V} = kr, \ \ddot{V} = k$

$V = mg r \cos\alpha$

$\dot{V} = mg \cos\alpha =$

$\ddot{V} = 0 = K$

?

$\boxed{f = \frac{1}{2\pi k}\sqrt{\frac{3L^2}{mr_0^4}}}$

ou

$V_{eff} = \frac{L^2}{2mr^2} + mg r \cos\alpha$

$\omega^2 = \frac{k}{m}$

$\dot{V} = -\frac{L^2}{mr^3} + mg\cos\alpha \rightarrow \ddot{V} = -\frac{L^2}{r_0^3} + mg\cos\alpha \Rightarrow \frac{3L^2}{mr_0^4} = K$

Q3.

a) r_0 e r_1 são os pontos de equilíbrio estável dos potenciais. $\left(\frac{dV}{dr} = 0\right)$

b) É esperado que a molécula B passe essa energia em [illegible], [illegible] e voltando para o estado fundamental [Inicialmente a molécula está oscilando em torno do ponto de equilíbrio. Quando ela muda para o 1º estado eletrônico a tendência é que a molécula pare de oscilar e [illegible] distância (aumentando r) ...] $r_1 < r_2$

c) $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ (1D) $\rightarrow P = \int |\psi(x)| dx$ (Valor 3D) $= \int |\psi(r)|^2 dV$

radial probability distribution is:

$P(r) = \frac{1}{\pi a_0^3} \cdot e^{-\frac{2r}{a_0}} \cdot \overbrace{4\pi r^2 \cdot dr}^{\frac{4}{3}\pi(r+dr)^3 - \frac{4}{3}\pi r^3}$

$1 - \frac{r}{a_0} \Rightarrow \boxed{r = a_0}$

$\frac{dP}{dr} = 0 = 8\pi r e^{-2r/a_0} - \frac{8\pi r^2}{a_0} e^{-2r/a_0}$

d) $\langle r \rangle = \int_0^\infty \Psi(r)^* \cdot r\, \Psi(r)\, dr = \frac{4\pi}{\pi a_0^3} \int_0^\infty r^3 \cdot e^{-\frac{2r}{a_0}} dr$

$u = r^3 \quad dv = e^{-\frac{2r}{a_0}}$

$du = 3r^2 dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

I

$r^3 \cdot \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \frac{3a_0}{2} e^{-\frac{2r}{a_0}} \cdot r^2\, dr$ (II)

$u = r^2 \quad dv = e^{-\frac{2r}{a_0}}$

$du = 2r\,dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

II

$r^2 \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \left(\frac{+a_0}{2}\right) e^{-\frac{2r}{a_0}} \cdot 2r\, dr$

$u = r \quad dv = e^{-\frac{2r}{a_0}}$

$du = dr \quad v = -\frac{a_0}{2} e^{-\frac{2r}{a_0}}$

III

$r \cdot \left(-\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} + \int \frac{+a_0}{2} e^{-\frac{2r}{a_0}} \cdot dr$

$= r\left(\frac{a_0}{2}\right) e^{-\frac{2r}{a_0}} - \frac{a_0^2}{2^2} e^{-\frac{2r}{a_0}}$

$\langle r \rangle = \frac{4}{a_0^3} \left\{ -\frac{r^3 a_0}{2} e^{-\frac{2r}{a_0}} + \frac{3a_0}{2} \left[-\frac{a_0 r^2}{2} e^{-\frac{2r}{a_0}} + a_0 \left(-\frac{a_0 r}{2} e^{-\frac{2r}{a_0}} - \left(\frac{a_0}{2}\right)^2 e^{-\frac{2r}{a_0}} \right) \right] \right\}_0^\infty$

$= \frac{4 e^{-\frac{2r}{a_0}}}{a_0^3} \left[-\frac{r^3 a_0}{2} - \frac{3a_0^2 r^2}{4} - \frac{3a_0^3 r}{4} - \frac{3a_0^4}{8} \right]_0^\infty$

$\boxed{\langle r \rangle = \frac{3a_0}{2}}$

Q4. a) Antes colisão. Depois colisão.

A v B C

m m M

A. $p = \gamma m v$. $\gamma = \frac{5}{3}$

$= m \cdot \frac{4}{5} c \cdot \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{4}{5} mc \cdot \frac{5}{3} = \boxed{\frac{4}{3} mc}$ ✓ $\boxed{E_A = \frac{5}{3} mc^2}$ ✓

B.

$p = 0$; $\boxed{E_B = m_0 c^2}$ ✓

C.

$\boxed{E = E_A + E_B = \frac{8}{3} m_0 c^2}$ ✓

$\boxed{p = \frac{4}{3} mc}$

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b) $\frac{v_c}{c^2} = \frac{p_c}{E_c} \Rightarrow v_c = \frac{\frac{4}{3} m c^3}{\frac{4}{3} m c^2} = \boxed{\frac{c}{2}}$

$p_c = \gamma m v_c = \frac{4}{3} m c$

$E_c = \gamma m c^2 = \frac{4}{3} m c^2$

$\gamma_1 = \frac{1}{\sqrt{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}}$

c) $\frac{4}{3} mc = \gamma M v_c \Rightarrow \frac{4}{3} m \cancel{c} = \frac{2}{\sqrt{3}} M \frac{\cancel{c}}{2} \Rightarrow M = \frac{4\sqrt{3}}{3} m \Rightarrow \boxed{M = \frac{4m}{\sqrt{3}}}$

Q5.

a) $dU = dQ - dW$ $\qquad p = \frac{nRT}{V}$

$nc\,dT = T dS - p dV$

$dS = \frac{nc\,dT}{T} - \frac{nR\,dV}{V} \Rightarrow \boxed{\Delta S = nc \ln\left(\frac{T_1}{T_1}\right) - nR \ln\left(\frac{V_1}{V_2}\right)}$

b) $A \to B$ $\qquad dU = dQ - dW \Rightarrow W = \Delta Q$

$\Delta U = 0$

$p_2 V_2 - p_1 V_1 = 0$

$p_2 V_2 = p_1 V_1$ $\qquad \frac{nR\Delta T}{V}$

$\boxed{W_1 = \int_{V_1}^{V_2} p\,dV = nRT_q \ln\left(\frac{V_2}{V_1}\right) = -Q_1}$

$Q = nc\Delta T + p V$ $\qquad \cancel{QW}$

$Q = n\Delta T (c_v + R)$

$\boxed{Q = n\Delta T\, c_p}$

$B \to C$ $\quad \Delta U = -W$

$\boxed{Q_2 = 0 \quad W_2 = nc\Delta T_1}$

$C \to D$

$W_3 = nRT_f \ln\left(\frac{V_4}{V_3}\right) = -Q_3$

$D \to A$

$Q_4 = 0 \quad W_4 = nc\Delta T_2$

$\Delta T_1 = T_f - T_q = -\Delta T_2$

c) $\eta = \frac{W_1}{Q_1} = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1} = \boxed{1 - \frac{T_f}{T_q}}$

↳ q/d do ciclo fechado

pelo facto que é convertido pl trabalho.

$\Delta U = 0 \Rightarrow W = \Delta Q = Q_1 - Q_3$

$\frac{Q_1}{Q_3} = \frac{T_q}{T_f}$

Q6

a) 

$r < R$

$E = \frac{Q_{int}}{2\pi r l \varepsilon_0} = \boxed{\frac{r\rho}{2\varepsilon_0}}$

$r > R$

$\boxed{E = \frac{R^2 \rho}{2r\varepsilon_0}}$

$Q_{int} = \pi R^2 l \rho$

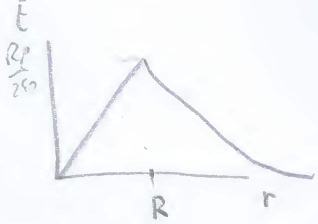
$Q_{int} = \pi r^2 l \rho$

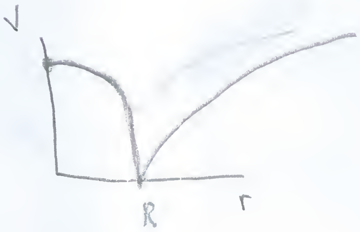
b) $r < R$

$V = \int_R^r -E\,dl = \int_R^r -\frac{r_1 \rho}{2\varepsilon_0}\,dr_1 = \underline{-\frac{\rho}{2\varepsilon_0} \cdot \left(\frac{r^2 - R^2}{2}\right)}$

$r > R$

$V = \int_R^r -\frac{R^2\rho}{2r_1\varepsilon_0}\,dr_1 = \frac{R^2\rho}{2\varepsilon_0}\left[\ln\left(\frac{r}{R}\right)\right]$

c) 



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d) Podemos aumentar a rigidez dielétrica do ar, ou seja, acrescentar um dielétrico na tubulação p/ aumentar K (constante dielétrica)

$$\boxed{E = \frac{Q_{enc}}{2\pi r l K \cdot \varepsilon_0}}$$

Q7.

a) $\vec{E} = E_0 \cdot e^{i(\vec{k}\vec{r} - \omega t)}$

$\boxed{\nabla \vec{E} = ik\vec{E}}$ $\boxed{\frac{\partial \vec{E}}{\partial t} = -i\omega\vec{E}}$

$H = E_0 \cdot e^{i(kz - \omega t)} \hat{y}$

$E = E_0 \cdot e^{i(k\hat{z} - \omega t)} \hat{x}$

$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

$ik \times \vec{E} - i\omega\vec{B} = 0$

$ik \cdot B = 0$

$ik D = \rho$

$ik \times H + i\omega D = J$

$ik \times H + i\omega\varepsilon\vec{E} = J$

b) $\rho = n$

$ik \times H + \frac{i}{k}\omega n = J$

$\frac{Le\vec{?}}{V} = n \quad \rho = \frac{Q}{V}$

$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ ik & ik & H \\ 0 & H & 0 \end{vmatrix} = ik\vec{H}\hat{z} - ikH\hat{x}$

$ik\vec{H}\hat{z} - ikH\hat{x} + \frac{i}{k}\omega n = J$

$J = \frac{i}{k}\omega n$

c)

Q8.

a) $\frac{\partial \rho}{\partial t} = -\frac{\partial j}{\partial x}$ $\qquad \frac{\partial \rho}{\partial t} = \frac{\partial[\psi^*(x,t)\cdot\psi(x,t)]}{\partial t} = \frac{\partial \psi^*(x,t)}{\partial t}\cdot\psi(x,t) + \psi^*(x,t)\cdot\frac{\partial \psi(x,t)}{\partial t}$

$= -\frac{\partial j}{\partial x}$

$$\boxed{i\hbar\cdot\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x,t)}$$

$$i\hbar\left(\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*(x,t)} - \frac{\partial \psi^*(x,t)}{\partial t}\cdot\frac{\psi(x,t)}{\psi^*(x,t)}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x,t)$$

Using

$$\boxed{-i\hbar\frac{\partial \psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + V\psi^*} \rightarrow \frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + \frac{iV\psi^*}{\hbar}$$

$$i\hbar\left(\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*} + \left(\frac{i\hbar}{2m}\cdot\frac{\partial^2 \psi^*}{\partial x^2} - \frac{iV\psi^*}{\hbar}\right)\cdot\frac{\psi}{\psi^*}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

$$i\hbar\frac{\partial \rho}{\partial t}\cdot\frac{1}{\psi^*} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \cancel{V(x)\psi(x)} + \frac{\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\cdot\frac{\psi}{\psi^*} - \cancel{V(x)}\psi$$

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2}\cdot\psi^* - \frac{i}{\hbar}\cancel{V(x)}\cdot\rho(x,t) - \frac{i\hbar}{2m}\frac{\partial^2 \psi^*}{\partial x^2}\cdot\psi + V(x)\cdot\cancel{\rho(x,t)}$$

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$$\boxed{\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m}\left[\frac{\partial^2 \Psi}{\partial x^2}\cdot\Psi^* - \frac{\partial^2 \Psi^*}{\partial x^2}\cdot\Psi\right]} \rightarrow \frac{\partial \Psi^*}{\partial x}\cdot\frac{\partial \Psi}{\partial x} + \Psi^*\cdot\frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2}\cdot\Psi - \frac{\partial \Psi^*}{\partial x}\cdot\frac{\partial \Psi}{\partial x}$$

b) $$-\frac{\partial J}{\partial x} = \frac{i\hbar}{2m}\underbrace{\left[\frac{\partial^2 \Psi}{\partial x^2}\Psi^* - \frac{\partial^2 \Psi^*}{\partial x^2}\cdot\Psi\right]}$$

$$-\frac{\partial J}{\partial x} = \frac{i\hbar}{2m}\frac{\partial}{\partial x}\left[\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\cdot\Psi\right]$$

$$\boxed{J = -\frac{i\hbar}{2m}\cdot\left[\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right]}$$

c) $\langle x\rangle = \int \Psi^*\cdot x\Psi\, dx \qquad \langle p\rangle = \int \Psi^*(-i\hbar)\cdot\frac{\partial \Psi}{\partial x}\, dx$

$$\frac{d\langle x\rangle}{dt} = \int \frac{\partial(\Psi^* x\Psi)}{\partial t}\, dx = \int\left[\frac{\partial \Psi^*}{\partial t}\cdot x\Psi + \Psi^*\cdot\frac{\partial x\Psi}{\partial t}\right] dx$$

$$= \int\left\{\left[\frac{-i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + \cancel{\frac{i}{\hbar}V\Psi^*}\right]\cdot x\Psi + \Psi^* x\left[\frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \cancel{\frac{i}{\hbar}V\Psi}\right]\right\} dx$$

$$= \int\left[\frac{-i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2}\cdot x\Psi + \frac{i\hbar}{2m}\Psi^* x\frac{\partial^2 \Psi}{\partial x^2}\right] dx = -\frac{i\hbar}{2m}\int\left(x\Psi\frac{\partial^2 \Psi^*}{\partial x^2} - x\Psi^*\frac{\partial^2 \Psi}{\partial x^2}\right) dx$$

$$\frac{d\langle x\rangle}{dt} = \frac{+i\hbar}{2m}\int x\left[-\psi\cdot\frac{\partial^2\psi^*}{\partial x^2} + \psi^*\cdot\frac{\partial^2\psi}{\partial x}\right]dx \qquad \langle p\rangle = \int \psi^*\cdot(-i\hbar)\frac{\partial\psi}{\partial x}dx$$

$$= \frac{-i\hbar}{2m}\int x\cdot\frac{\partial}{\partial x}\left[\psi^*\frac{\partial\psi}{\partial x} - \psi\cdot\frac{\partial\psi^*}{\partial x}\right]dx$$

$u = x \quad dv = \frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)$

$du = dx$

$v = \psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}$

$$= \frac{-i\hbar}{2m}\left[\underbrace{\cancel{\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)\cdot x}\Big|_{-\infty}^{\infty}}_{0} - \int \psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\,dx\right]$$

$\left[\psi\frac{\partial\psi}{\partial x} \to \infty \text{ nu aphi gen } \frac{1}{x}\right]$

$\left[\frac{1}{x}\frac{1}{x}\cdot x\right] = \frac{1}{x}$

$$\Rightarrow \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)dx$$

$$= \frac{1}{2m}\left(\langle p\rangle + \int_{-\infty}^{\infty} i\hbar\,\psi\frac{\partial\psi^*}{\partial x}dx\right) = \frac{1}{2m}\left(\langle p\rangle + \langle p\rangle\right) = \boxed{\frac{\langle p\rangle}{m}}$$

$u = \psi \quad du = \frac{\partial\psi}{\partial x}$ $dv = \frac{\partial\psi^*}{\partial x} i\hbar \quad v = i\hbar\psi^*$

$= \cancel{i\hbar\psi^*\psi}^{\,0} - \int_{-\infty}^{\infty} i\hbar\psi^*\frac{\partial\psi}{\partial x}dx = \langle p\rangle$

Q5.

a) $\hat{N}\hat{a}|n\rangle = \hat{a}^\dagger\hat{a}\cdot\hat{a}|n\rangle = \left[\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\right]\hat{a}|n\rangle$

$= \left[[\hat{a}^\dagger,\hat{a}] + \hat{a}\hat{a}^\dagger\right]\hat{a}|n\rangle = -\hat{a}|n\rangle + \hat{a}n|n\rangle = (n-1)\hat{a}|n\rangle$

autostate! autovalor $\Rightarrow n-1 = n'$

$\hat{N}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\cdot\hat{a}\cdot\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\cdot\left[\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\right]|n\rangle$

$= \hat{a}^\dagger\left[1 + \hat{a}^\dagger\hat{a}\right]|n\rangle = \underline{(n+1)\hat{a}^\dagger|n\rangle}$, autovalor $\Rightarrow n+1 = n''$

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b) $\boxed{\langle n|n\rangle = 1}$ $\boxed{\hat{a}|n\rangle = c_n|n'\rangle}$

$n = \langle n|N|n\rangle = \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \langle \hat{a}n|\hat{a}n\rangle$

$\sqrt{n}\cdot\sqrt{n+1}$

$\sqrt{n(n+1)} = n$

$n^2 = n(n+1)$

$n = n+1$

$\langle n|\hat{a}^\dagger\hat{a}|n\rangle = c_n\langle n|\hat{a}^\dagger|n-1\rangle$

$\langle n|n|n\rangle = n\cdot 1$

$c_n\langle n-1|\hat{a}|n\rangle^* = |c_n|^2$

$\langle n-1|n-1\rangle = 1$

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ $n \geq 0$ $|c|^2 \geq 0$

$|c_n|^2 = n$

$\langle H\rangle = \frac{\langle p^2\rangle}{2m} + \langle \frac{1}{2}m\omega^2 x^2\rangle \geq 0$ $\hat{a}|0\rangle = 0$

$\boxed{c_n = \sqrt{n}}$

c) $\hat{N}\hat{a}|n\rangle = \hat{a}^\dagger\hat{a}\cdot\hat{a}|n\rangle = (\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle = (\{\hat{a}^\dagger,\hat{a}\} - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle$

$(1 - \hat{a}\hat{a}^\dagger)\hat{a}|n\rangle = -\hat{a}|n\rangle - \hat{a}\cdot n|n\rangle = \boxed{(-n+1)\hat{a}|n\rangle}$

$\boxed{n' = -n+1}$

$\hat{N}\hat{a}^\dagger|n\rangle = \hat{a}^\dagger\hat{a}\cdot\hat{a}^\dagger|n\rangle = \hat{a}^\dagger(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a})|n\rangle$

$= \hat{a}^\dagger(\{\hat{a},\hat{a}^\dagger\} - \hat{a}^\dagger\hat{a})|n\rangle = \hat{a}^\dagger(1 - \hat{a}^\dagger\hat{a})|n\rangle = \hat{a}^\dagger(1-n)|n\rangle = \boxed{(1-n)\hat{a}^\dagger|n\rangle}$

$\boxed{n' = (1-n)}$

$\hat{a}|n\rangle = c_n|-n-1\rangle$

$n = \langle n|N|n\rangle = \langle n|\hat{a}^\dagger\hat{a}|n\rangle = c_n\langle n|\hat{a}^\dagger|-n-1\rangle$

$= c_n\langle -n-1|\hat{a}|n\rangle^* = |c_n|^2\langle -n-1|-n-1\rangle$

$\boxed{\hat{a}|n\rangle = \sqrt{n}|n'\rangle}$

$c_n = \sqrt{n}$

d) auto-valor real

$\sqrt{1-n} \geq 0 \Rightarrow n \leq 1$ $\Rightarrow$ $\boxed{n = 0 \text{ e } n = 1}$

$\sqrt{n} \geq 0 \Rightarrow n \geq 0$

$\boxed{4}$

Q10

a) $u(T) = aT^4$

$U = u(T).V$ e $S = s(T).V$, $P = \frac{u(T)}{3}$

$dU = dF - dW$ $\quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$dU = TdS - pdV$

$\frac{dU}{dV} = T\left(\frac{\partial S}{\partial V}\right)_T - P$

$u(T) = T\left(\frac{\partial P}{\partial T}\right)_V - P$

$u(T) = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V - \frac{u(T)}{3}$ $\Rightarrow 3u(T) + u(T) = \frac{T}{3}\frac{\partial u}{\partial T}$

$\frac{dT}{T} = \frac{du}{4u(T)}$

$\boxed{u(T) = AT^4}$

b) ii $u = \int_0^\infty \hbar\omega \cdot g(\omega)\, d\omega$

n° fotões por vol. e energia de cada

$u = \hbar \int_0^\infty n \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1} = \frac{\hbar}{(\beta\hbar)^4} \cdot \int_0^\infty n \frac{v^3 dv}{e^v - 1} = \frac{\pi^4 n}{15\hbar^3} \cdot \frac{k^4 T^4}{} = \sigma T^4$

$v = \beta\hbar\omega \quad dv = \beta\hbar$

b) $Z = \sum_n \left(e^{-\beta\varepsilon_j}\right)^n = \frac{1}{1 - e^{-\beta\varepsilon_j}} \Rightarrow \ln Z = -\ln\left(1 - e^{-\beta\varepsilon_j}\right)$

$\bar{n}_j = +\frac{1}{\beta} \cdot \left[\frac{1}{1 - e^{-\beta\varepsilon_j}} \cdot \left(-e^{-\beta\varepsilon_j} \cdot (-\beta)\right)\right] = \frac{+e^{-\beta\varepsilon_j}}{1 - e^{-\beta\varepsilon_j}} = \frac{+1}{e^{\beta\varepsilon_j} - 1}$